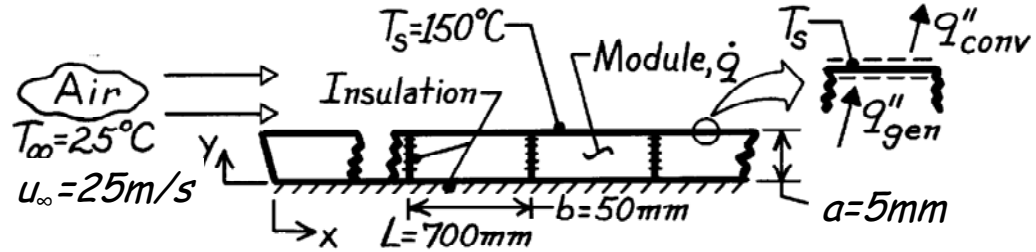


PROBLEM 7.8

KNOWN: Flat plate comprised of rectangular modules of surface temperature T_s , thickness a and length b cooled by air at 25°C and a velocity of 25 m/s . Prescribed thermophysical properties of the module material.

FIND: (a) Required power generation for the module positioned 700 mm from the leading edge of the plate and (b) Maximum temperature in this module.

SCHEMATIC:



ASSUMPTIONS: (1) Laminar flow at leading edge of plate, (2) Transition Reynolds number of 5×10^5 , (3) Heat transfer is one-dimensional in y -direction within each module, (4) $\dot{q}$ is uniform within module, (5) Negligible radiation heat transfer.

PROPERTIES: Module material (given): $k = 5.2\text{ W/m}\cdot\text{K}$, $c_p = 320\text{ J/kg}\cdot\text{K}$, $\rho = 2300\text{ kg/m}^3$; Table A-4, Air ($\bar{T}_f = (T_s + T_\infty)/2 = 360\text{ K}$, 1 atm): $k = 0.0308\text{ W/m}\cdot\text{K}$, $\nu = 22.02 \times 10^{-6}\text{ m}^2/\text{s}$, $\text{Pr} = 0.698$.

ANALYSIS: (a) The module power generation follows from an energy balance on the module surface,

$$q_{\text{conv}}'' = q_{\text{gen}}''$$

$$\bar{h}(T_s - T_\infty) = \dot{q} \cdot a \quad \text{or} \quad \dot{q} = \frac{\bar{h}(T_s - T_\infty)}{a}.$$

To select a convection correlation for estimating $\bar{h}$, first find the Reynolds numbers at $x = L$.

$$\text{Re}_L = \frac{u_\infty L}{\nu} = \frac{25\text{ m/s} \times 0.70\text{ m}}{22.02 \times 10^{-6}\text{ m}^2/\text{s}} = 7.947 \times 10^5.$$

Since the flow is turbulent over the module, the approximation $\bar{h} \approx h_x (L + b/2)$ is appropriate, with

$$\text{Re}_{L+b/2} = \frac{25\text{ m/s} \times (0.700 + 0.050/2)\text{ m}}{22.02 \times 10^{-6}\text{ m}^2/\text{s}} = 8.231 \times 10^5.$$

Using the turbulent flow correlation with $x = L + b/2 = 0.725\text{ m}$,

$$\text{Nu}_x = \frac{h_x x}{k} = 0.0296 \text{Re}_x^{4/5} \text{Pr}^{1/3}$$

$$\text{Nu}_x = 0.0296 (8.231 \times 10^5)^{4/5} (0.698)^{1/3} = 1418$$

$$\bar{h} \approx h_x = \frac{\text{Nu}_x k}{x} = \frac{1418 \times 0.0308\text{ W/m}\cdot\text{K}}{0.725} = 60.2\text{ W/m}^2 \cdot \text{K}.$$

Continued ...

PROBLEM 7.8 (Cont.)

Hence,

$$\dot{q} = \frac{60.2 \text{ W/m}^2 \cdot \text{K} (150 - 25) \text{ K}}{0.005 \text{ m}} = 1.51 \times 10^6 \text{ W/m}^3. \quad <$$

(b) The maximum temperature within the module occurs at the surface next to the insulation ($y = 0$). For one-dimensional conduction with thermal energy generation, use Eq. 3.42 to obtain

$$T(0) = \frac{\dot{q} a^2}{2k} + T_s = \frac{1.51 \times 10^6 \text{ W/m}^3 \times (0.005 \text{ m})^2}{2 \times 5.2 \text{ W/m} \cdot \text{K}} + 150^\circ \text{C} = 153.6^\circ \text{C}. \quad <$$

COMMENTS: An alternative approach for estimating the average heat transfer coefficient for the module follows from the relation

$$\begin{aligned} \bar{q}_{\text{module}} &= \bar{q}_{0 \rightarrow L+b} - \bar{q}_{0 \rightarrow L} \\ \bar{h} \cdot b &= \bar{h}_{L+b} \cdot (L+b) - \bar{h}_L \cdot L \quad \text{or} \quad \bar{h} = \bar{h}_{L+b} \frac{L+b}{b} - \bar{h}_L \frac{L}{b}. \end{aligned}$$

Recognizing that laminar and turbulent flow conditions exist, the appropriate correlation is

$$\overline{\text{Nu}}_x = \left(0.037 \text{Re}_x^{4/5} - 871 \right) \text{Pr}^{1/3}$$

With $x = L + b$ and $x = L$, find

$$\bar{h}_{L+b} = 43.05 \text{ W/m}^2 \cdot \text{K} \quad \text{and} \quad \bar{h}_L = 41.82 \text{ W/m}^2 \cdot \text{K}.$$

Hence,

$$\bar{h} = \left[43.05 \frac{0.750}{0.050} - 41.82 \frac{0.700}{0.05} \right] \text{ W/m}^2 \cdot \text{K} = 60.3 \text{ W/m}^2 \cdot \text{K}.$$

which is in excellent agreement with the approximate result employed in part (a).